

Chapter Twenty-six
VERTICAL ALIGNMENT

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Chapter Twenty-six

VERTICAL ALIGNMENT

The highway vertical alignment plays a significant role in a highway's safety, aesthetics and project costs. Chapter Twelve of the Montana Road Design Manual provides numerical criteria for various vertical alignment elements. Chapter Twenty-six provides additional guidance on these and other vertical alignment elements, including critical lengths of grade, climbing lanes, vertical curvature computations and vertical clearances.

26.1 DEFINITIONS/NOMENCLATURE

1. Bus. A heavy vehicle involved in the transport of passengers.
2. Critical Length of Grade. The maximum length of a specific upgrade on which a loaded truck can operate without experiencing a specified reduction in speed.
3. Gradient. The rate of slope between two adjacent vertical points of intersection (VPI) expressed as a percent. The numerical value for percent of grade is the vertical rise or fall in feet (meters) for each 100 ft (100 m) of horizontal distance. Upgrades in the direction of stationing are identified as plus (+). Downgrades are identified as minus (-).
4. Heavy Vehicles. Any vehicle with more than four wheels touching the pavement during normal operation. Heavy vehicles collectively include trucks, recreational vehicles and buses.
5. K-Values. The horizontal distance needed to produce a 1% change in gradient.
6. Level Terrain. Level terrain is generally considered to be flat, and has minimal impact on vehicular performance. Highway sight distances are either long or could be made long without major construction expense.
7. Momentum Grade. A site where an upgrade is preceded by a downgrade, thereby allowing a truck to increase its speed on the upgrade. This increase in speed allows the designer to use a higher speed reduction in the critical length of grade figure.
8. Mountainous Terrain. Longitudinal and transverse changes in elevation are abrupt. Benching and side hill excavation are frequently required to provide the

desirable highway alignment. Mountainous terrain aggravates the performance of trucks relative to passenger cars, resulting in some trucks operating at crawl speeds.

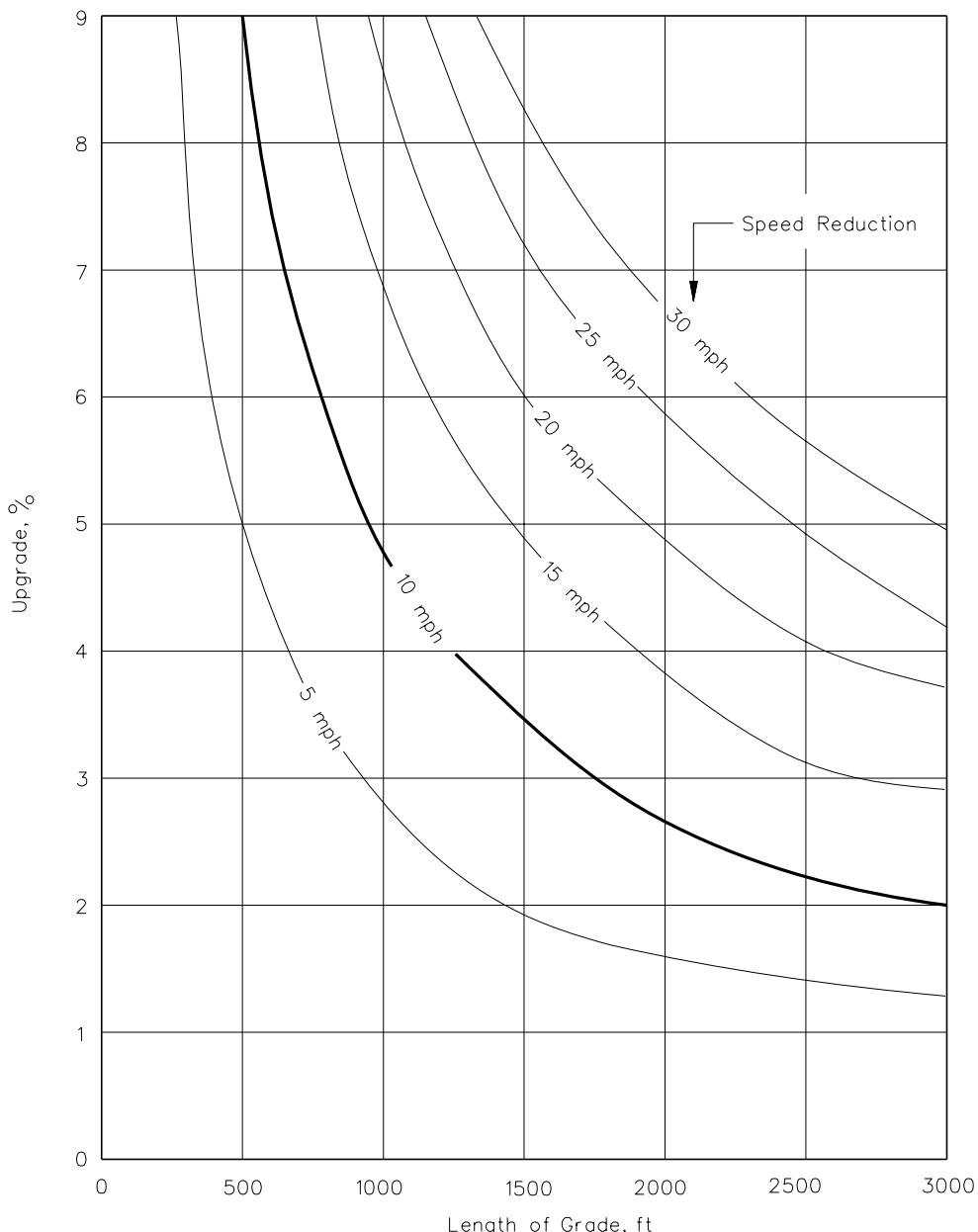
9. Performance Curves. A set of curves that illustrate the effect grades will have on the design vehicle's acceleration and/or deceleration.
10. Profile Grade Line. A series of tangent lines connected by vertical curves. It is typically placed along the roadway centerline of undivided facilities and at the edges of the two roadways on the median side on divided facilities. Chapter Ten of the Montana Road Design Manual discusses those factors which should be evaluated when establishing the profile grade line.
11. Recreational Vehicle. A heavy vehicle, generally operated by a private motorist, engaged in the transportation of recreational equipment or facilities; examples include campers, boat trailers, motorcycle trailers, etc.
12. Rolling Terrain. The natural slopes consistently rise above and fall below the roadway grade and, occasionally, steep slopes present some restriction to the desirable highway alignment. In general, rolling terrain generates steeper grades, causing trucks to reduce speeds below those of passenger cars.
13. Spline Curve. A curve drawn using a flexible template to meet field conditions.
14. Truck. A heavy vehicle engaged primarily in the transport of goods and materials, or in the delivery of services other than public transportation. For geometric design and capacity analyses, trucks are defined as vehicles with six or more tires. Data on trucks are compiled and reported by the Rail, Transit and Planning Division.
15. Vertical Point of Curvature (VPC). The point at which a tangent grade ends and the vertical curve begins.
16. Vertical Point of Intersection (VPI). The point where the extension of two tangent grades intersect.
17. Vertical Point of Tangency (VPT). The point at which the vertical curve ends and the tangent grade begins.

26.2 GRADES

26.2.1 Critical Length of Grade

Critical length of grade is the maximum length of a specific upgrade on which a loaded truck can operate without experiencing a specified reduction in speed. The highway gradient in combination with the length of grade will determine the truck speed reduction on upgrades. The following will apply to the critical length of grade:

1. Design Vehicle. For critical-length-of-grade determinations, the Department has adopted the 200 lb/hp (120 kilograms/kilowatt (kg/kW)) truck as the most representative design vehicle for Montana.
2. Criteria. [Figure 26.2A](#) provides the critical lengths of grade for a given percent grade and acceptable truck speed reduction. Although these figures are based on an initial truck speed of 70 mph (110 km/h), they apply to any design speed. For design purposes, use the 10 mph (15 km/h) speed reduction curve to determine if the critical length of grade is exceeded.
3. Momentum Grades. Where an upgrade is preceded by a downgrade, trucks will often increase speed to make the climb. A speed increase of 5 mph (10 km/h) on moderate downgrades (-3% to -5%) and 10 mph (15 km/h) on steeper downgrades (greater than -5%) of sufficient length are reasonable adjustments. These can be used in design by using a higher speed reduction curve in [Figure 26.2A](#) (i.e., use the 15 mph (25 km/h) speed reduction curve for downgrades of -3% to -5% and the 20 mph (30 km/h) speed reduction curve for downgrades greater than -5%). However, the designer should consider that these speed increases may not be attainable if traffic volumes are high enough that a truck may be behind a passenger vehicle when descending the momentum grade. Therefore, these increases in speed should only be considered if the highway has a LOS B or better.
4. Measurement. Vertical curves are part of the length of grade. [Figure 26.2B](#) illustrates how to measure the length of grade to determine the critical length of grade from [Figure 26.2A](#).
5. Highway Types. The critical-length-of-grade criteria applies equally to rural 2-lane or multi-lane highways.
6. Application. If the critical length of grade is exceeded, the designer should either flatten the grade, if practical, or should evaluate the need for a truck-climbing lane, see [Section 26.2.2](#).

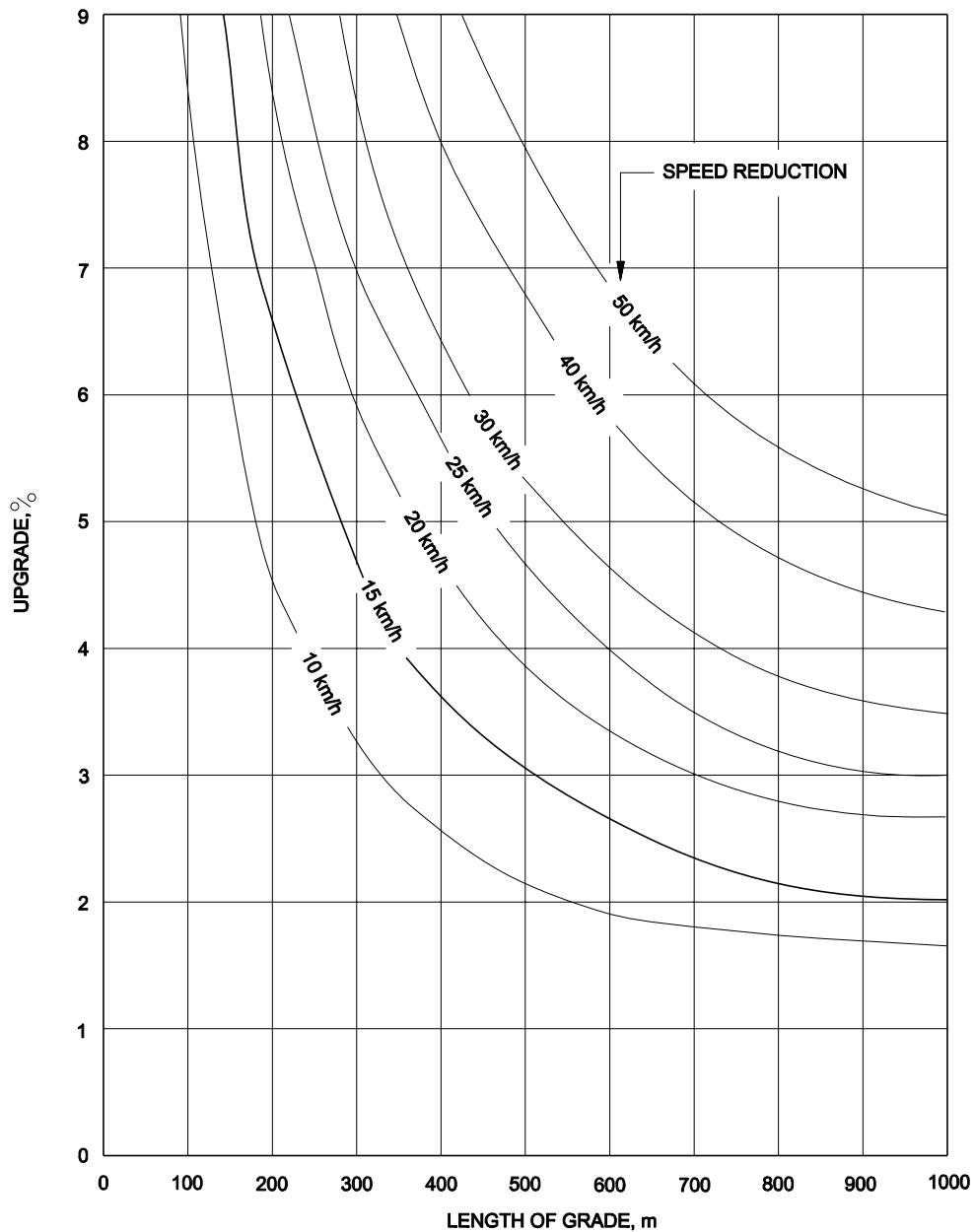


Notes:

1. Typically, the 10 mph curve will be used.
2. Where an upgrade is preceded by a downgrade and if the highway has a LOS B or better, the 15 mph speed reduction curve may be used with preceding downgrades of 3% - 5%, or the 20 mph speed reduction curve may be used with preceding downgrades greater than 5%.
3. Figure based on a truck with initial speed of 70 mph. However, it may be used for any design speed.

**CRITICAL LENGTH OF GRADE
(200 lb/hp Truck) (US Customary)**

Figure 26.2A

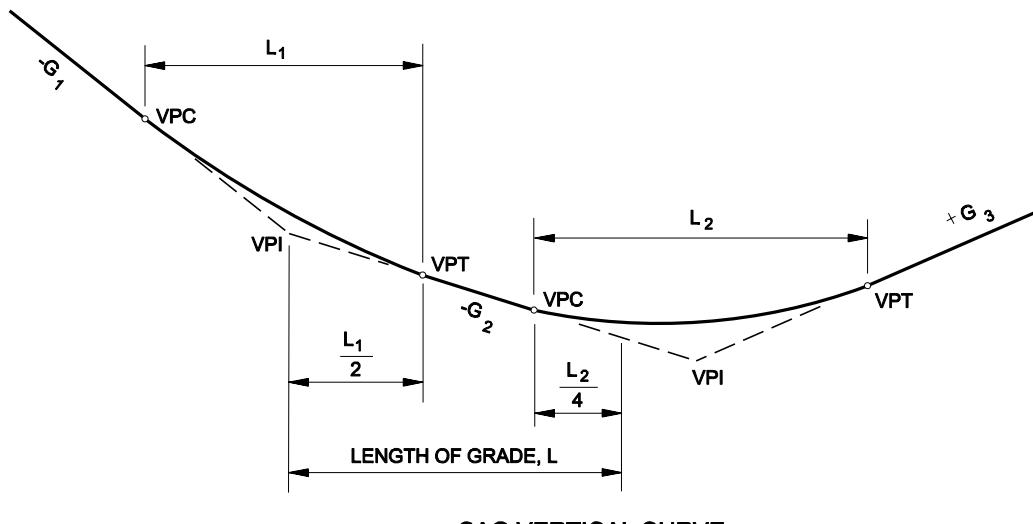
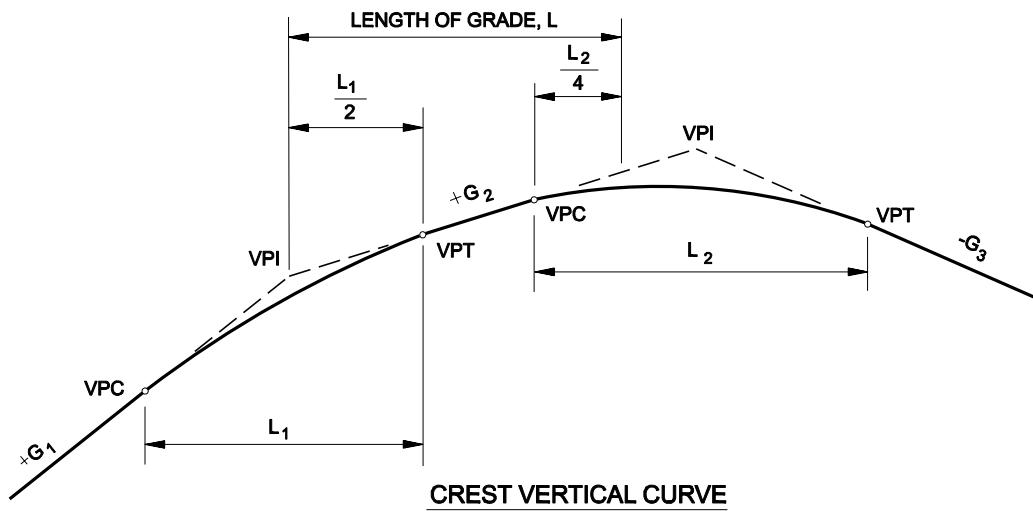


Notes:

1. Typically, the 15 km/h curve will be used.
2. Where an upgrade is preceded by a downgrade and if the highway has a LOS B or better, the 25 km/h speed reduction curve may be used with preceding downgrades of 3% - 5%, or the 30 km/h speed reduction curve may be used with preceding downgrades greater than 5%.
3. Figure based on a truck with initial speed of 110 km/h. However, it may be used for any design speed.

**CRITICAL LENGTH OF GRADE
(120 Kg/kW Truck) (Metric)**

Figure 26.2A



Notes:

1. For vertical curves where the two tangent grades are in the same direction (both upgrades or both downgrades), 50% of the curve length will be part of the length of grade.
2. For vertical curves where the two tangent grades are in opposite directions (one grade up and one grade down), 25% of the curve length will be part of the length of grade.
3. The above diagram is included for illustrative purposes only. Broken-back curves are to be avoided wherever practical.

MEASUREMENT FOR LENGTH OF GRADE
Figure 26.2B

* * * * *

Example 26.2-1

Given: Level Approach

$G = +4\%$

$L = 1500$ ft (length of grade)

Rural Arterial

Problem: Determine if the critical length of grade is exceeded.

Solution: [Figure 26.2A](#) yields a critical length of grade of 1200 ft for a 10 mph speed reduction. The length of grade (L) exceeds this value. Therefore, the designer should flatten the grade, if practical, or evaluate the need for a climbing lane.

Example 26.2-2

Given: [Figure 26.2C](#) illustrates the vertical alignment on a low-volume, 2-lane rural minor arterial.

Problem: Determine if the critical length of grade is exceeded for G_2 or for the combination upgrade G_3 and G_4 .

Solution: Use the following steps:

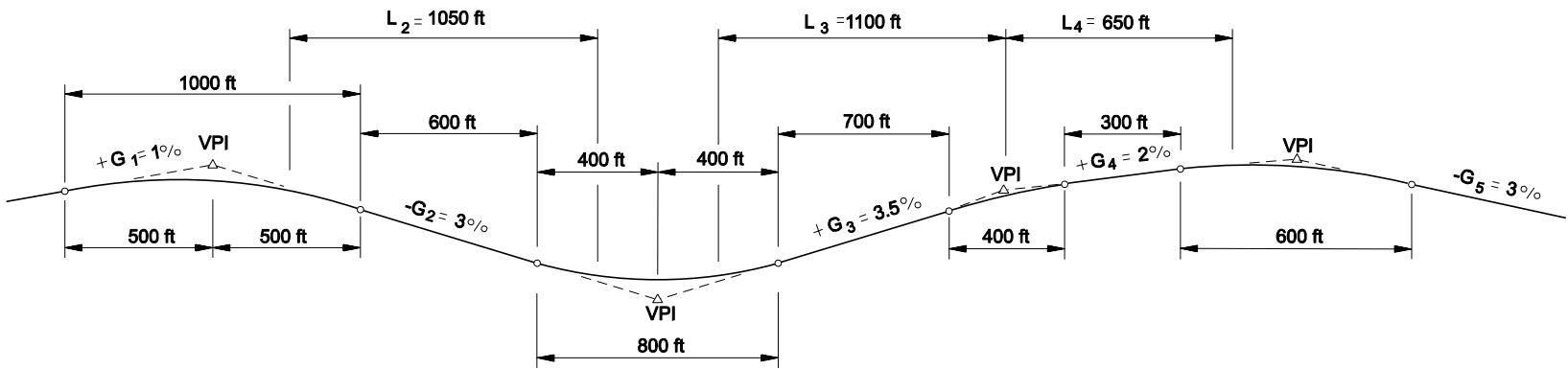
Step 1. Determine the length of grade using the criteria in [Figure 26.2B](#). For this example, these are calculated as follows:

$$L_2 = \frac{1000}{4} + 600 + \frac{800}{4} = 1050 \text{ ft}$$

$$L_3 = \frac{800}{4} + 700 + \frac{400}{2} = 1100 \text{ ft}$$

$$L_4 = \frac{400}{2} + 300 + \frac{600}{4} = 650 \text{ ft}$$

Step 2. Determine the critical length of grade in both directions. Use [Figure 26.2A](#) to determine the critical length of grade:



CRITICAL LENGTH OF GRADE CALCULATIONS
(Example 26.2-2)

Figure 26.2C

- a. For trucks traveling left to right, enter into [Figure 26.2A](#) the value for G_3 (3.5%) and $L_3 = 1050$ ft. The speed reduction is 7.5 mph. For G_4 (2%) and $L_4 = 650$ ft, the speed reduction is approximately 3.5 mph. The total speed reduction on the combination upgrade G_3 and G_4 is 11 mph. This exceeds the maximum 10 mph speed reduction. However, on low-volume roads, one can assume a 5 mph increase in truck speed for the 3% "momentum" grade (G_2) which precedes G_3 . Therefore, a speed reduction may be as high as 16 mph before concluding that the combination grade exceeds the critical length of grade. Assuming the benefits of the momentum grade, this leads to the conclusion that the critical length of grade is not exceeded.
- b. For trucks traveling in the opposite direction, on [Figure 26.2A](#), enter in the value for G_2 (3%) and determine that the critical length of grade for the 10 mph speed reduction is 1700 ft. Because L_2 is less than 1700 ft (i.e., 1050 ft), the critical length of grade for this direction is not exceeded.

* * * * *

26.2.2 Truck-Climbing Lanes

The Traffic Engineering Section typically reviews the need for truck-climbing lanes and provides the design details for the lanes where they are necessary. This applies to either a project administered by the Road Design Section or a geometric design project administered by the Traffic Engineering Section.

26.2.2.1 Guidelines

Guidelines for truck-climbing lanes are primarily based on a highway capacity analysis. [Chapter Thirty](#) presents the capacity analysis, which is an adaptation of the methodology presented in the [Highway Capacity Manual](#) and the [AASHTO A Policy on Geometric Design of Highways and Streets](#).

26.2.2.2 Design

[Figure 26.2D](#) summarizes the design details for a truck-climbing lane. The designer should also consider the following:

DESIGN ELEMENT	DESIRABLE	MINIMUM
Lane Width	12 ft (3.6 m)	12 ft (3.6 m)
Shoulder Width	Same as approach roadway.	4 ft (1.2 m) for a 32 ft (9.6 m) or wider approach roadway. 2 ft (0.6 m) for a 28 ft (8.4 m) wide approach roadways.
Cross Slope on Tangent	2%	2%
Beginning of Full-Width Lane (1)	Near the VPC of the vertical curve preceding the grade.	To where the truck speed has been reduced to 10 mph (15 km/h) below the design speed.
End of Full-Width Lane (2)	To where truck has reached 10 mph (15 km/h) below highway design speed	To where truck has reached 10 mph (15 km/h) below highway design speed.
Entering Taper	25:1	150 ft (50 m)
Exiting Taper	(3)	50:1
Minimum Full-Width Length	N/A	1200 ft (365 m)

Notes:

- (1) Use [Figure 26.2F](#) to determine truck deceleration rates.
- (2) Use [Figure 26.2F](#) to determine truck acceleration rates. Also, see [Comment #4](#) in Section 26.2.2.2.
- (3) Use the following exiting taper rates:

Design Speed		Exiting Taper Rate
US Customary (mph)	Metric (km/h)	
50	80	50:1
55	90	55:1
60	100	60:1
70	110	70:1
75	120	75:1

DESIGN CRITERIA FOR TRUCK-CLIMBING LANES

Figure 26.2D

1. **Design Speed.** For entering speeds equal to or greater than 55 mph (90 km/h), use 55 mph (90 km/h) for the truck design speed. For speeds less than 55 mph (90 km/h), use the roadway design speed. Under restricted conditions, the designer may want to consider the effect a momentum grade will have on the entering speed. See [Section 26.2.1](#) for additional information on momentum grades. However, the maximum speed used for design should be 55 mph (90 km/h).
2. **Superelevation.** For horizontal curves, the truck-climbing lane will be superelevated at the same rate as the adjacent travel lane.
3. **Performance Curves.** [Figure 26.2E](#) presents the deceleration and acceleration rates for a 200 lb/hp (120 kg/KW) truck. To properly determine the location of the beginning and end of the full-width climbing lane, the designer will typically need to develop a truck speed profile. [Section 26.2.2.3](#) provides information on how to develop a truck speed profile.
4. **End of Full-Width Lane.** In addition to the criteria in [Figure 26.2D](#), the designer should consider the available sight distance to the point where the truck will merge back into the through travel lane. At a minimum, this should meet the criteria presented in [Figure 26.2E](#). The sight distances in [Figure 26.2E](#) are the decision sight distances, avoidance maneuver C, from the [AASHTO A Policy on Geometric Design of Highways and Streets](#). Desirably, the driver should have significantly more sight distance available to the merge point at the end of the taper to safely complete the maneuver, especially where the merge is on a horizontal or vertical curve.

US Customary		Metric	
Design Speed (mph)	Minimum Sight Distance (ft)	Design Speed (km/h)	Minimum Sight Distance (m)
50	750	80	230
55	865	90	270
60	990	100	315
70	1105	110	330
75	1180	120	360

MINIMUM SIGHT DISTANCE FOR ENDING TRUCK-CLIMBING LANES

Figure 26.2E

5. Traffic Control Devices. [Chapter Nineteen](#) presents the Department's typical application for pavement markings and highway signing for truck-climbing lanes.

26.2.2.3 Truck Speed Profile

For highways with a single grade, the critical length of grade and deceleration and acceleration rates can be directly determined from [Figure 26-2F](#). However, most highways have a continuous series of grades. Often, it is necessary to find the impact of a series of significant grades in succession. If several different grades are present, then a speed profile may need to be developed.

The following example illustrates how to construct a truck speed profile and how to use [Figure 26.2F](#).

* * * * *

Example 26.2-3

Given: Level Approach

$G_1 = +3\%$ for 800 ft (VPI to VPI)

$G_2 = +5\%$ for 3200 ft (VPI to VPI)

$G_3 = -2\%$ beyond the composite upgrade (G_1 and G_2)

$V = 60$ mph (design speed)

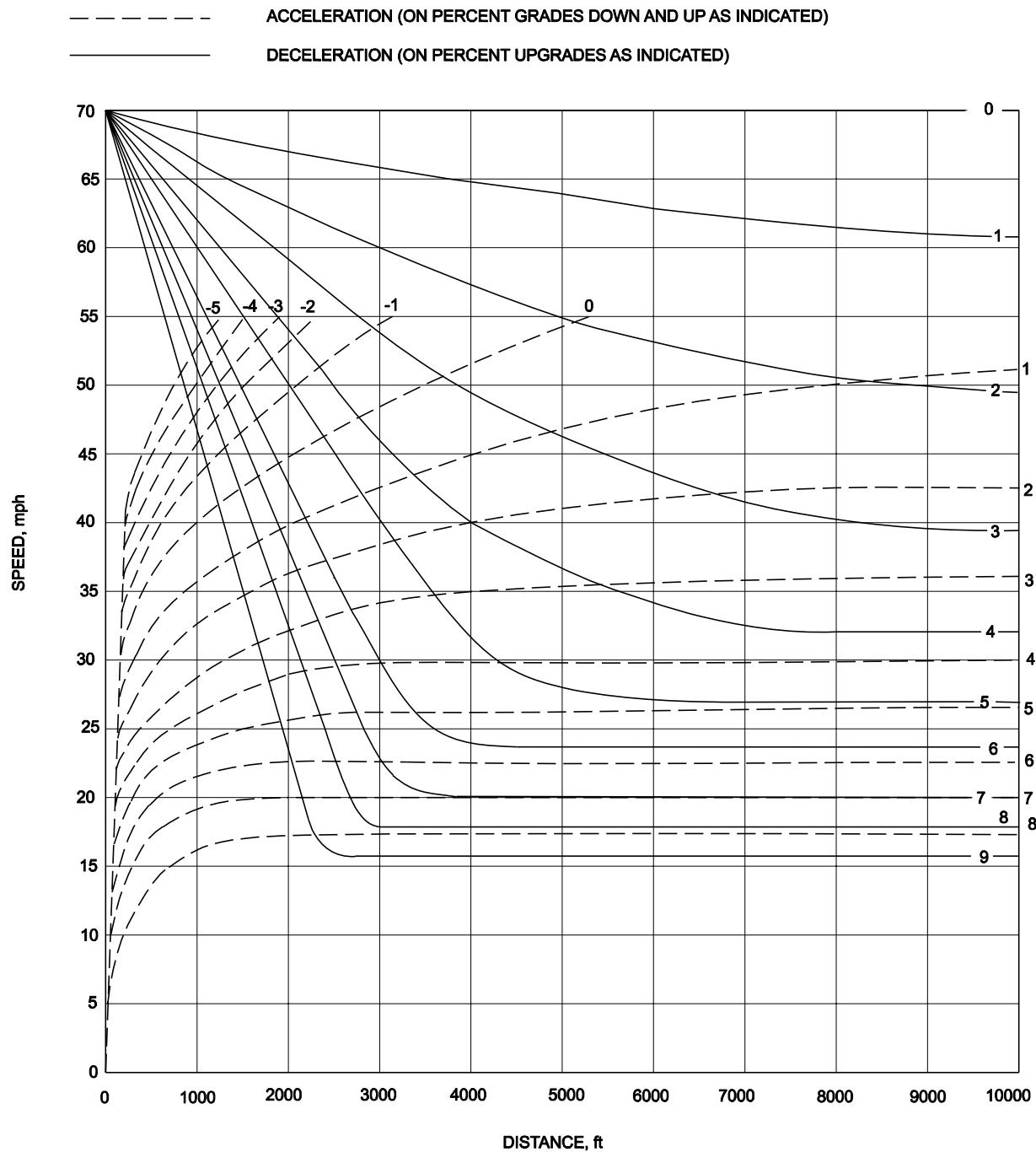
Rural Arterial

Problem: Using the criteria in [Figure 26.2D](#) and [Figure 26.2F](#), construct a truck speed profile and determine the beginning and ending points of the full-width climbing lane.

Solution: The following steps apply:

Step 1: Determine the truck speed on G_1 , at 200 ft increments, using [Figure 26.2F](#) and plot them in [Figure 26.2G](#). Assume an initial truck speed of 55 mph (see [Figure 26.2F](#)).

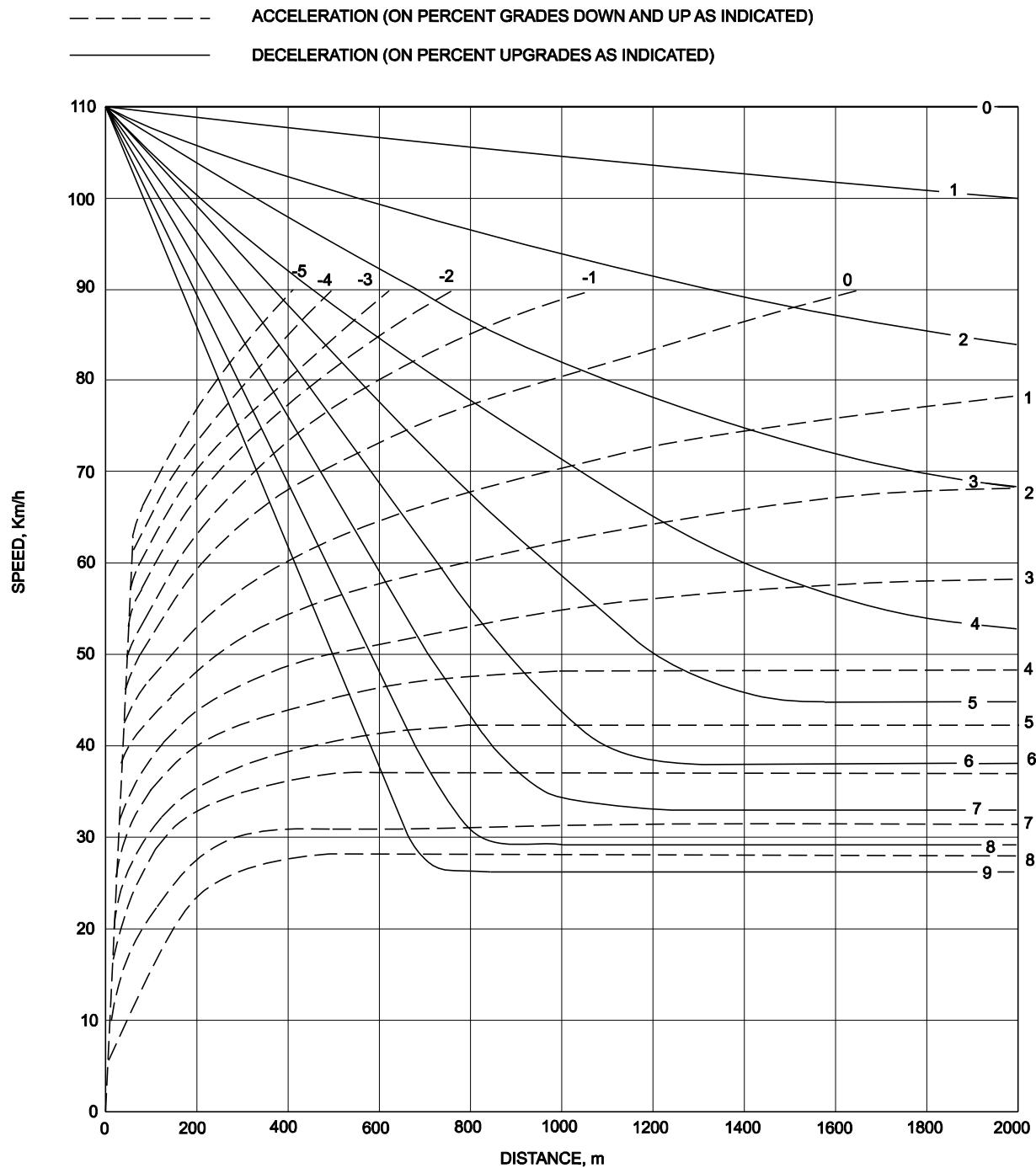
Distance From VPI ₁ (ft)	Horizontal Distance on Figure 26.2F (ft)	Truck Speed (mph)	Comments
0	2800	55	
200	3000	54	VPI ₁
400	3200	53	
600	3400	52	
800	3600	51	VPI ₂



Note: For design speeds above 70 mph, use an initial speed of 70 mph. For design speeds 70 mph and below, use the design speed as the initial speed.

PERFORMANCE CURVES FOR LARGE TRUCKS (200 lb/hp) (US Customary)

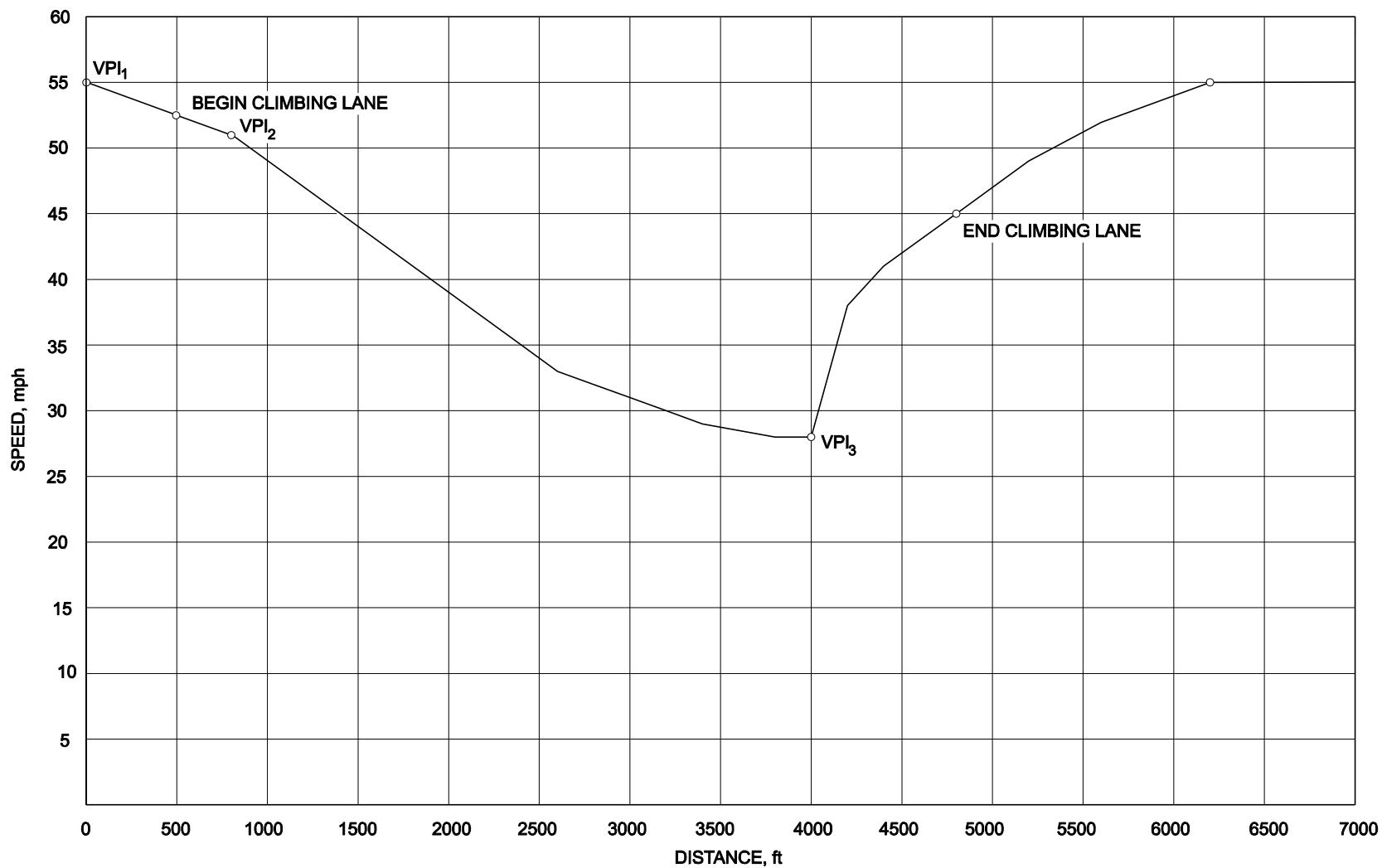
Figure 26.2F



Note: For design speeds above 110 km/h, use an initial speed of 110 km/h. For design speeds 110 km/h and below, use the design speed as the initial speed.

PERFORMANCE CURVES FOR LARGE TRUCKS (120 kg/kW) (Metric)

Figure 26.2F



TRUCK SPEED PROFILE
(Example 26.2-3)

Figure 26.2G

Step 2: Determine the truck speed on G_2 , at 200 ft increments, using [Figure 26.2F](#) and plot them in [Figure 26.2G](#). From Step 1, the initial speed on G_2 is the final speed from G_1 (i.e., 51 mph). Move right horizontally along the 51 mph line to the 5% deceleration curve. This is approximately 1900 ft along the horizontal axis. This is the starting point for G_2 .

Distance From VPI ₁ (ft)	Horizontal Distance on Figure 26.2F (ft)	Truck Speed (mph)	Comments
800	1900	51	
1000	2100	49	
1200	2300	47	
1400	2500	45	
1600	2700	43	
1800	2900	41	
2000	3100	39	
2200	3300	37	
2400	3500	35	
2600	3700	33	
2800	3900	32	
3000	4100	31	
3200	4300	30	
3400	4500	29	
3600	4700	29	
3800	4900	28	
4000	5100	28	VPI ₃

Step 3: Determine the truck speed on G_3 , at 200 ft increments using [Figure 26.2F](#) until the truck has fully accelerated to 55 mph and plot them in [Figure 26.2G](#). The truck will have a speed of 28 mph as it enters the 2% downgrade at VPI₃. Read into [Figure 26.2F](#) at the 28 mph point on the vertical axis move over horizontally to the -2% line. This is approximately 150 ft along the horizontal axis. This is the starting point for G_3 .

Distance From VPI ₁ (ft)	Horizontal Distance on Figure 26.2F (ft)	Truck Speed (mph)	Comments
4000	150	28	
4200	350	38	
4400	550	41	
4600	750	43	
4800	950	45	
5000	1150	47	
5200	1350	49	
5400	1550	50	
5600	1750	52	
5800	1950	53	
6000	2150	54	
6200	2350	55	

Step 4: Determine the beginning and end of the full-width climbing lane. From [Figure 26.2D](#), the beginning of the full-width lane will begin near the VPC of the vertical curve proceeding the grade. For ending the full-width climbing lane, the design criteria from [Figure 26.2D](#) is where the truck speed has reached a speed of 45 mph (10 mph below the design speed). This occurs at 4800 ft beyond VPI₁.

26.3 VERTICAL CLEARANCES

Figure 26.3A summarizes the minimum vertical clearances for various highway classifications and conditions.

Type	Minimum Clearance	
	US Customary	Metric
Freeway Under	17'-0" ⁽¹⁾	5.20 m ⁽¹⁾
Arterial Under	17'-0" ⁽¹⁾	5.20 m ⁽¹⁾
Collector Under	16'-6" ⁽¹⁾	5.05 m ⁽¹⁾
Roadway under Pedestrian Bridge	17'-6"	5.35 m
Roadway under Traffic Signal	17'-6" ^{(1) (2)}	5.35 m ^{(1) (2)}
Railroad under Roadway (Typical)	23'-3½" ⁽³⁾	7.10 m ⁽³⁾
Roadway under Sign Truss	17'-6" ⁽¹⁾	5.35 ⁽¹⁾

Notes:

1. *Value allows 6 in (150 mm) for future resurfacing.*
2. *Distance is measured from roadway surface to the bottom of signal at the bottom of the back plate.*
3. *Contact the Utilities Section and the Bridge Bureau to determine the allowable railroad vertical clearance.*

MINIMUM VERTICAL CLEARANCES

Figure 26.3A

26.4 VERTICAL CURVES

26.4.1 Crest Vertical Curves

Crest vertical curves are in the shape of a parabola. The basic equations for determining the minimum length of a crest vertical curve are:

$$L = \frac{AS^2}{200(\sqrt{h_1} + \sqrt{h_2})^2} \quad (\text{Equation 26.4-1})$$

$$L = KA \quad (\text{Equation 26.4-2})$$

Where:

L	=	length of vertical curve, ft (m)
A	=	algebraic difference between the two tangent grades, %
S	=	sight distance, ft (m)
h_1	=	height of eye above road surface, ft (m)
h_2	=	height of object above road surface, ft (m)
K	=	horizontal distance needed to produce a 1% change in gradient

The length of the crest vertical curve will depend upon "A" for the specific curve and upon the selected sight distance, height of eye and height of object. The following discusses the selection of these values. For design purposes, the calculated length of curve based on the rounded K-value should be rounded up to the next highest 50 ft (20 m) increment.

The principal control in the design of crest vertical curves is to ensure that, at a minimum, stopping sight distance (SSD) is available throughout the curve. [Figure 26.4A](#) presents the K-values for stopping sight distance. The following discusses the application of the K-values:

1. Passenger Cars (Level Grade). [Figure 26.4A](#) presents K-values for passenger cars. These are calculated by assuming $h_1 = 3.5$ ft (1.080 m), $h_2 = 2$ ft (0.6 m) and $S = SSD$ in the basic equation for crest vertical curves (Equation 26.4-1). The values represent the lowest acceptable sight distance on a facility. Where cost effective, the designer should strive to provide a design in which the K-values exceeds the SSD.

As discussed in [Section 24.7](#), a design exception is needed if the vertical curve does not provide the SSD for level conditions.

US Customary				Metric			
Design Speed (mph)	Rounded ⁽¹⁾ SSD for Design (ft)	Rate of Vertical Curvature, K ⁽²⁾		Design Speed (km/h)	Rounded ⁽¹⁾ SSD for Design (m)	Rate of Vertical Curvature, K ⁽³⁾ Design	
		Calculated	Design			Calculated	Design
20	115	6.1	7	30	35	1.9	2
25	155	11.1	12	40	50	3.8	4
30	200	18.5	19	50	65	6.4	7
35	250	29.0	29	60	85	11.0	11
40	305	43.1	44	70	105	16.8	17
45	360	60.1	61	80	130	25.7	26
50	425	83.7	84	90	160	38.9	39
55	495	113.5	114	100	185	52.0	52
60	570	150.6	151	110	220	73.6	74
65	645	192.8	193	120	250	95.0	95
70	730	246.9	247				
75	820	311.6	312				

Notes:

1. Stopping sight distances (SSD) are from [Figure 24.5A](#).
2.
$$K = \frac{SSD^2}{2158}, \text{ where: } h_1 = 3.5 \text{ ft}, h_2 = 2 \text{ ft} \quad (\text{US Customary})$$
3.
$$K = \frac{SSD^2}{658}, \text{ where: } h_1 = 1.080 \text{ m}, h_2 = 0.6 \text{ m} \quad (\text{Metric})$$

K-VALUES FOR CREST VERTICAL CURVES (Level Grades)

Figure 26.4A

2. Minimum Length. For aesthetics, the suggested minimum length of a crest vertical curve on a rural highway is 1000 ft (300 m). For small values of A, the calculated curve lengths may actually be zero. However, angle points are not allowed on rural highways. Therefore, the minimum length of curve is based on Equation 26.4-3.

$$L_{\min} = 3 V \quad (\text{US Customary}) \quad (\text{Equation 26.4-3})$$

$$L_{\min} = 0.6 V \quad (\text{Metric}) \quad (\text{Equation 26.4-3})$$

Where:

$$\begin{aligned} L_{\min} &= \text{minimum length of vertical curve, ft (m)} \\ V &= \text{design speed, mph (km/h)} \end{aligned}$$

3. Drainage. Drainage should be considered in the design of crest vertical curves where curbed sections are used. Drainage problems should not be experienced if the vertical curvature is sharp enough so that a minimum longitudinal grade of at least 0.3% is reached at a point about 50 ft (15 m) from either side of the apex. To ensure that this objective is achieved, the length of the vertical curve should be based upon a K-value of 167 (51) or less. For crest vertical curves on curbed sections where this K-value is exceeded, the drainage design should be more carefully evaluated near the apex.

For uncurbed sections of highway, drainage should not be a problem at crest vertical curves. However, the adjacent roadside ditch should have minimum longitudinal gradient of 0.2% in the vicinity of the vertical curve.

4. Passing Sight Distance. At some locations, it may be desirable to provide passing sight distance in the design of crest vertical curves. On rural reconstruction projects, the designer should attempt to provide passing sight distance over as much of the highway length as practical. It will generally not be cost effective, however, to make significant improvements to the horizontal and vertical alignment solely to increase the available passing sight distance. Section 24.5.2 discusses the application and design values for passing sight distance. These "S" values are used in the basic equations for crest vertical curves ([Equations 26.4-1](#) and [26.4-2](#)). The height of eye (h_1) is 3.5 ft (1.080 m) and the height of object (h_2) is 3.5 ft (1.080 m). [Figure 26.4B](#) presents the minimum K-values for determining passing sight distance. The pavement markings for the no-passing zones will be based on the criteria presented in [Chapter Nineteen](#).

US Customary			Metric		
Design Speed (mph)	Passing ⁽¹⁾ Sight Distance (ft)	Rate of Vertical Curvature, K ⁽²⁾ Design	Design Speed (km/h)	Passing ⁽¹⁾ Sight Distance (m)	Rate of Vertical Curvature, K ⁽³⁾ Design
30	1090	424	50	345	138
35	1280	585	60	410	195
40	1470	772	70	485	272
45	1625	943	80	540	338
50	1835	1203	90	615	438
55	1985	1407	100	670	520
60	2135	1628	110	730	617
65	2285	1865	120	775	695
70	2480	2197			
75	2580	2377			

Notes:

1. Passing sight distances are from [Figure 24.5B](#).
2.
$$K = \frac{PSD^2}{2800}, \text{ where: } h_1 = 3.5 \text{ ft}, h_2 = 3.5 \text{ ft} \quad (\text{US Customary})$$
3.
$$K = \frac{PSD^2}{864}, \text{ where: } h_1 = 1.080 \text{ m}, h_2 = 1.080 \text{ m} \quad (\text{Metric})$$

K-VALUES FOR PASSING SIGHT DISTANCE (Crest Vertical Curves)

Figure 26.4B

26.4.2 Sag Vertical Curves

Sag vertical curves are in the shape of a parabola. Typically, they are designed to allow the vehicular headlights to illuminate the roadway surface (i.e., the height of object = 0.0 ft (0.0 m)) for a given distance "S." These assumptions yield the following basic equations for determining the minimum length of sag vertical curves:

$$L = \frac{AS^2}{200h_3 + 3.5S} \quad (\text{Equation 26.4-4})$$

$$L = KA \quad (\text{Equation 26.4-5})$$

Where:

- L = length of vertical curve, ft (m)
- A = algebraic difference between the two tangent grades, %
- S = sight distance, ft (m)
- h_3 = height of headlights above pavement surface, ft (m)
- K = horizontal distance needed to produce a 1% change in gradient

The length of the sag vertical curve will depend upon "A" for the specific curve and upon the selected sight distance and headlight height. For design purposes, the calculated length of curve based on the rounded K-value should be rounded up to the next highest 50 ft (20 m) increment.

The principal control in the design of sag vertical curves is to ensure that at a minimum, stopping sight distance (SSD) is available for headlight illumination throughout the curve. The design assumes that there is a 1° upward divergence of the light beam from the longitudinal axis of the headlights. [Figure 26.4C](#) presents the K-values for stopping sight distances. The following discusses the application of the K-values:

1. Passenger Cars. [Figure 26.4C](#) presents K-values for passenger cars. These are calculated by assuming $h_3 = 2$ ft (600 mm) and $S = \text{SSD}$ in the basic equation for sag vertical curves (Equation 26.4-4). These values represent the lowest acceptable sight distance on a facility. However, the designer should strive to provide a design in which the K-values exceed SSD.

As discussed in [Section 24.7](#), a design exception is needed if the vertical curve does not provide the SSD for level conditions.

US Customary				Metric			
Design Speed (mph)	Stopping Sight Distance (ft)	Rate of Vertical Curvature, K ⁽²⁾		Design Speed (km/h)	Stopping Sight Distance (m)	Rate of Vertical Curvature, K ⁽³⁾	
		Calculated (ft)	Design (ft)			Calculated (m)	Design (m)
20	115	16.5	17	30	35	5.1	6
25	155	25.5	26	40	50	8.5	9
30	200	36.4	37	50	65	12.2	13
35	250	49.0	49	60	85	17.3	18
40	305	63.4	64	70	105	22.6	23
45	360	78.1	79	80	130	29.4	30
50	425	95.7	96	90	160	37.6	38
55	495	114.9	115	100	185	44.6	45
60	570	135.7	136	110	220	54.4	55
65	645	156.5	157	120	250	62.8	63
70	730	180.3	181				
75	820	205.6	206				

Notes:

1. Stopping sight distances (SSD) are from [Figure 24-5A](#).
2.
$$K = \frac{SSD^2}{400 + 3.5SSD}, \text{ where: } h_3 = 2 \text{ ft} \quad (\text{US Customary})$$
3.
$$K = \frac{SSD^2}{120 + 3.5SSD}, \text{ where: } h_3 = 0.6 \text{ m} \quad (\text{Metric})$$

K-VALUES FOR SAG VERTICAL CURVES

Figure 26.4C

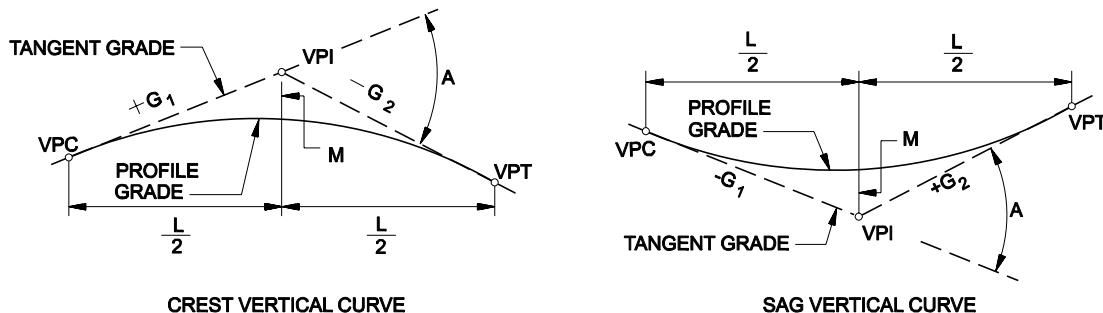
2. **Minimum Length**. For most sag vertical curves, the minimum length of curve should be based on [Equation 26.4-3](#) (i.e., $L_{min} = 3V$ ($L_{min} = 0.6V$)). For aesthetics, the suggested minimum length of a sag vertical curve on a rural highway is 1000 ft (300 m).
3. **Drainage**. Drainage should be considered in the design of sag vertical curves where curbed sections are used. Drainage problems are minimized if the sag vertical curve is sharp enough so that a minimum longitudinal grade of at least 0.3% is reached at a point about 50 ft (15 m) from either side of the low point. To achieve this objective, the length of the vertical curve should be based upon a K-value of 167 (51) or less. For sag vertical curves on curbed sections where this K-value is exceeded, the drainage design should be more carefully evaluated near the low point. For example, it may be necessary to install flanking inlets on either side of the low point.

26.4.3 Vertical Curve Computations

The following will apply to the mathematical design of vertical curves:

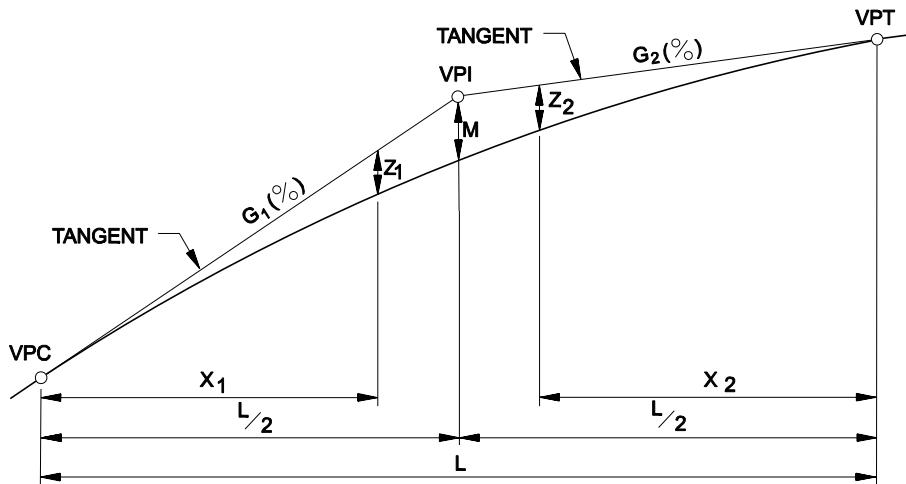
1. **Definitions**. [Figure 26.4D](#) presents the common terms and definitions used in vertical curve computations.
2. **Measurements**. All measurements for vertical curves are made on the horizontal or vertical plane, not along the profile grade. With the simple parabolic curve, the vertical offsets from the tangent vary as the square of the horizontal distance from the VPC or VPT. Elevations along the curve are calculated as proportions of the vertical offset at the point of vertical intersection (VPI). The necessary formulas for computing the vertical curve are shown in [Figure 26.4E](#). [Figure 26.4F](#) provides an example of how to use these formulas.
3. **Unsymmetrical Vertical Curve**. Occasionally, it is necessary to use an unsymmetrical vertical curve to obtain clearance on a structure or to meet other field conditions. This curve is similar to the parabolic vertical curve, except the curve does not vary symmetrically about the VPI. The necessary formulas for computing the unsymmetrical vertical curve are shown in [Figure 26.4G](#).
4. **Vertical Curve Through Fixed Point**. A vertical highway curve often must be designed to pass through an established point. For example, it may be necessary to tie into an existing transverse road or to clear existing structures. See [Figure 26.4H](#). [Figure 26.4I](#) illustrates an example on how to use these formulas.

ELEMENT	ABBREVIATION	DEFINITION
Vertical Point of Curvature	VPC	The point at which a tangent grade ends and the vertical curve begins.
Vertical Point of Tangency	VPT	The point at which the vertical curve ends and the tangent grade begins.
Vertical Point of Intersection	VPI	The point where the extension of two tangent grades intersect.
Grade	G_1, G_2	The rate of slope between two adjacent VPI's expressed as a percent. The numerical value for percent of grade is the vertical rise or fall in feet (meters) for each 100 ft (100 m) of horizontal distance. Upgrades in the direction of stationing are identified as plus (+). Downgrades are identified as minus (-).
External Distance	M	The vertical distance (offset) between the VPI and the roadway surface along the vertical curve.
Algebraic Difference in Grade	A	The value of A is determined by the deflection in percent between two tangent grades ($G_2 - G_1$).
Length of Vertical Curve	L	The horizontal distance in feet (meters) from the VPC to the VPT.
Tangent Elevation	Tan. Elev.	The elevation on the tangent line between the VPC and VPI and the VPI and VPT.
Elevation on Vertical Curve	Curve Elev.	The elevation of the vertical curve at any given point along the curve.
Horizontal Distance	X	Horizontal distance measured from the VPC or VPT to any point on the vertical curve, in feet (meters).
Tangent Offset	Z	Vertical distance from the tangent line to any point on the vertical curve, in feet (meters).
Low/High Point	x_T	The station at the high point for crest curves or the low point for sag curves. At this point, the slope of the tangent to the curve is equal to 0%.
Symmetrical Curve	—	The VPI is located at the mid-point between VPC and VPT stationing
Unsymmetrical Curve	—	The VPI is not located at the mid-point between VPC and VPT stationing.



VERTICAL CURVE DEFINITIONS

Figure 26.4D



M = External distance, ft (m)

Z = Any tangent offset, ft (m)

L = Horizontal length of vertical curve, ft (m)

X = Horizontal distance from VPC or VPT to any ordinate "Z," ft (m)

G₁ & G₂ = Rates of grade, expressed algebraically, %

NOTE: ALL EXPRESSIONS TO BE CALCULATED ALGEBRAICALLY —
(Use algebraic signs of grades; grades in percent.)

1. Elevations of VPC and VPI:

$$\text{ELEV. OF VPC} = \text{ELEV. VPI} - G_1 \left(\frac{L}{200} \right) \quad (\text{Equation 26.4-6})$$

$$\text{ELEV. OF VPT} = \text{ELEV. VPI} + G_2 \left(\frac{L}{200} \right) \quad (\text{Equation 26.4-7})$$

2. For the elevation of any point "X" on the vertical curve:

$$\text{CURVE ELEV.} = \text{TAN ELEV.} + Z \quad (\text{Equation 26.4-8})$$

Where:

Left of VPI (x₁ measured from VPC):

$$(a) \text{ TAN ELEV.} = \text{VPC ELEV.} + G_1 \left(\frac{X_1}{100} \right)$$

$$(b) Z_1 = X_1^2 \frac{(G_2 - G_1)}{200 L}$$

Right of VPI (x₂ measured from VPT):

$$(a) \text{ TAN ELEV.} = \text{VPT ELEV.} - G_2 \left(\frac{X_2}{100} \right)$$

$$(b) Z_2 = X_2^2 \frac{(G_2 - G_1)}{200 L}$$

3. Calculating high or low point in the vertical curve:

$$(a) \text{ To determine distance "x}_T\text{" from VPC:} \quad X_T = \frac{L G_1}{G_1 - G_2} \quad (\text{Equation 26.4-9})$$

$$(b) \text{ To determine high or low point stationing:} \quad \text{VPC Sta.} + x_T \quad (\text{Equation 26.4-10})$$

(c) To determine high or low point elevation on a vertical curve:

$$\text{ELEV.}_{\text{HIGH OR LOW POINT}} = \text{VPC ELEV.} - \frac{L G_1^2}{(G_2 - G_1) 200} \quad (\text{Equation 26.4-11})$$

SYMMETRICAL VERTICAL CURVE EQUATIONS

Figure 26.4E

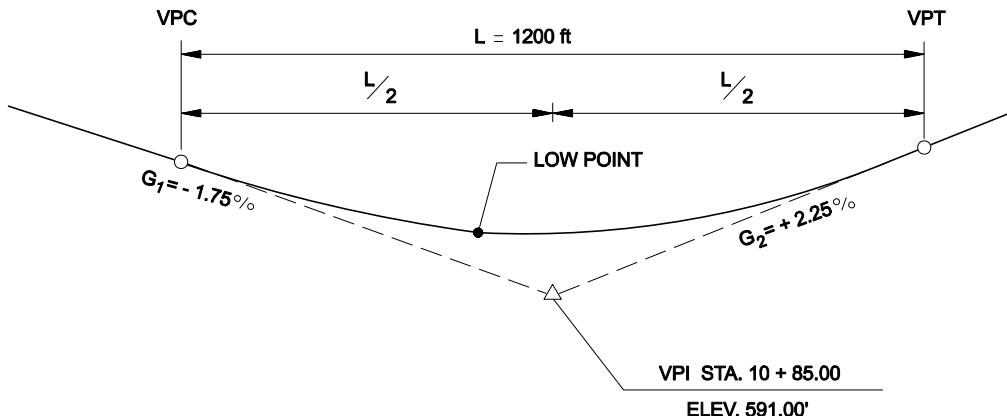
Example 26-4.1

Given: $G_1 = -1.75\%$
 $G_2 = +2.25\%$
 Elev. of VPI = 591 ft
 Station of VPI = 10 + 85.00
 $L = 1200$ ft
 Symmetrical Vertical Curve

Problem: Compute the vertical curve elevations for each 100 ft station. Compute the low point elevation and stationing.

Solution:

1. Draw a diagram of the vertical curve and determine the stationing at the beginning (VPC) and the end (VPT) of the curve.



$$\text{VPC Station} = \text{VPI Sta} - \frac{1}{2}L = (10 + 85) - (600) = 4 + 85.00$$

$$\text{VPT Station} = \text{VPI Sta} + \frac{1}{2}L = (10 + 85) + (600) = 16 + 85.00$$

2. Vertical curve equations:

$$\text{CURVE ELEV.} = \text{TAN ELEV.} + Z \quad (\text{Equation 26.4-8})$$

Where:

Left of VPI (x₁ measured from VPC):

$$(a) \text{ TAN ELEV.} = \text{VPC ELEV.} + G_1 \left(\frac{X_1}{100} \right)$$

$$(b) \ Z_1 = X_1^2 \frac{(G_2 - G_1)}{200 L}$$

Right of VPI (x₂ measured from VPT):

$$(a) \text{ TAN ELEV.} = \text{VPT ELEV.} - G_2 \left(\frac{X_2}{100} \right)$$

$$(b) \ Z_2 = X_2^2 \frac{(G_2 - G_1)}{200 L}$$

3. Set up a table to show the vertical curve elevations at the 100 ft stations, substituting the values into the above equations.

VERTICAL CURVE COMPUTATIONS (Example 26.4-1)

Figure 26.4F

Example 26.4-1

Solution: (continued)

Station	Control Point	Tangent Elevation (ft)	x	x^2	$z = x^2/60000$	Grade Elevation (ft)
4+85	VPC	601.50	0	0	0.00	601.50
5+85		599.75	100	10000	0.17	599.92
6+85		598.00	200	40000	0.67	598.67
7+85		596.25	300	90000	1.50	597.75
8+85		594.50	400	160000	2.67	597.17
9+85		592.75	500	250000	4.17	596.92
10+85		591.00	600	360000	6.00	597.00
11+85		593.25	500	250000	4.17	597.42
12+85		595.50	400	160000	2.67	598.17
13+85		597.75	300	90000	1.50	599.25
14+85		600.00	200	40000	0.67	600.67
15+85		602.25	100	10000	0.17	602.42
16+85	VPT	604.50	0	0	0.00	604.50

4. Calculate low point (Equations 26.4-9, 26.4-10 and 26.4-11):

$$X_T = \frac{LG_1}{G_1 - G_2} = \frac{1200(-1.75)}{-1.75 - 2.25} = \frac{-2100}{-4.00} = 525 \text{ ft from VPC}$$

therefore, the Station at low point is:

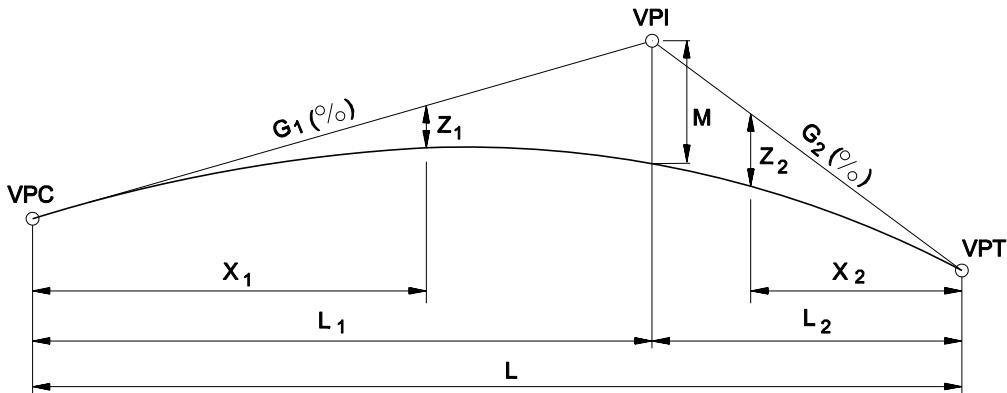
$$VPC_{STA} + X_T = (4 + 85) + (525) = 10 + 10.00$$

elevation of low point on curve equals:

$$\text{Elev. VPC} - \frac{LG_1^2}{G_2 - G_1, 200} = 601.50 - \frac{1200(-1.75)^2}{(2.25 - (-1.75)) 200} = 601.50 - 4.59 = 596.91 \text{ ft}$$

VERTICAL CURVE COMPUTATIONS
(Example 26.4-1) (continued)

Figure 26.4F



M = Offset from the VPI to the curve (external distance), ft (m)

Z = Any tangent offset, ft (m)

L = Horizontal length of vertical curve, ft (m)

L_1 = Horizontal distance from VPC to VPI, ft (m)

L_2 = Horizontal distance from VPT to VPI, ft (m)

X = Horizontal distance from VPC or VPT to any ordinate "Z," ft (m)

G_1 & G_2 = Rates of grade, expressed algebraically, %

NOTE: ALL EXPRESSIONS TO BE CALCULATED ALGEBRAICALLY

(Use algebraic signs of grades; grades in percent.)

1. Elevations of VPC and VPT:

$$\text{ELEV. OF VPC} = \text{ELEV. VPI} - G_1 \left(\frac{L_1}{100} \right) \quad (\text{Equation 26.4-12})$$

$$\text{ELEV. OF VPT} = \text{ELEV. VPI} + G_2 \left(\frac{L_2}{100} \right) \quad (\text{Equation 26.4-13})$$

2. For the elevation of any point "X" on the vertical curve:

$$\text{CURVE ELEV.} = \text{TAN ELEV.} + Z \quad (\text{Equation 26.4-14})$$

Where:

Left of VPI (x_1 measured from VPC):

$$(a) \text{ TAN ELEV.} = \text{VPC ELEV.} + G_1 \left(\frac{X_1}{100} \right)$$

$$(b) \ Z_1 = X_1^2 \left(\frac{L_2}{L_1} \right) \left(\frac{(G_2 - G_1)}{200 L} \right)$$

Right of VPI (x_2 measured from VPT):

$$(a) \text{ TAN ELEV.} = \text{VPT ELEV.} - G_2 \left(\frac{X_2}{100} \right)$$

$$(b) \ Z_2 = X_2^2 \left(\frac{L_1}{L_2} \right) \left(\frac{(G_2 - G_1)}{200 L} \right)$$

UNSYMMETRICAL VERTICAL CURVE EQUATIONS

Figure 26.4G

3. Calculating High or Low Point on Curve:

Note: Two answers will be determined by solving the equations below. Only one answer is correct. The incorrect answer is where $X_T > L_1$ on the left side of the VPI or where $X_T > L_2$ on the right side of the VPI.

a. Assume high or low point occurs left of VPI to determine the distance, X_T , from VPC:

$$X_T = \frac{L_1}{L_2} \left[\frac{G_1 L}{(G_1 - G_2)} \right] \quad (\text{Equation 26.4-15})$$

Note: Does $X_T > L_1$? If yes, this answer is incorrect and the high or low point is on the right side of the VPI. (Go to Step d. to solve for the high or low point elevation.) If no, then this is the correct answer and proceed with Steps b. and c. below.)

b. To determine high or low point stationing (where $X_T < L_1$):

$$STA_{HIGH \text{ OR } LOW \text{ POINT}} = VPC \text{ STA.} + X_T \quad (\text{Equation 26.4-16})$$

c. To determine high or low point elevation on vertical curve (when $X_T < L_1$):

$$ELEV._{HIGH \text{ OR } LOW \text{ POINT}} = ELEV. \text{ VPC} - \frac{L_1}{L_2} \left[\frac{LG_1^2}{(G_2 - G_1)(200)} \right] \quad (\text{Equation 26.4-17})$$

d. If $X_T > L_1$ from Step a., the higher or low point occurs right of the VPI. Determine the distance X_T from the VPT:

$$X_T = \frac{L_2}{L_1} \left[\frac{G_2 L}{(G_2 - G_1)} \right] \quad (\text{Equation 26.4-18})$$

e. To determine high or low point stationing:

$$STA_{HIGH \text{ OR } LOW \text{ POINT}} = VPT \text{ STA.} - X_T \quad (\text{Equation 26.4-19})$$

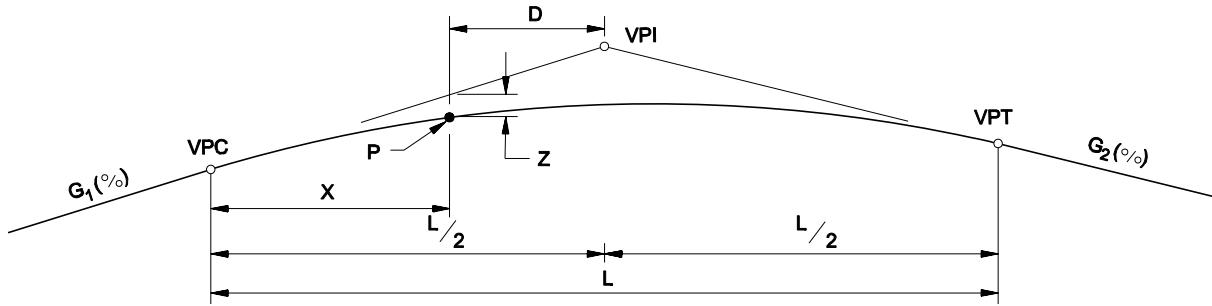
f. To determine high or low point elevation on the vertical curve:

$$ELEV._{HIGH \text{ OR } LOW \text{ POINT}} = ELEV. \text{ VPT} - \frac{L_2}{L_1} \left[\frac{LG_2^2}{(G_2 - G_1)(200)} \right] \quad (\text{Equation 26.4-20})$$

UNSYMMETRICAL VERTICAL CURVE EQUATIONS

(Continued)

Figure 26.4G



TO PASS A SYMMETRICAL VERTICAL CURVE THROUGH A GIVEN POINT (P)

G_1 = Grade in, %

D = Distance from "P" to VPI, ft (m)

G_2 = Grade out, %

L = Length of vertical curve, ft (m)

A = Algebraic difference in grades, %

y = Vertical curve correction at point "P", ft (m)

x = Distance from VPC to "P", ft (m)

Given: G_1, G_2, D

Find: Length of vertical curve.

Solution:

1. Find algebraic difference in grades:

$$A = G_2 - G_1$$

2. Find vertical curve correction at Point P:

From [Equation 26.4-8](#), (x measured from VPC):

$$Z = X^2 \left(\frac{G_2 - G_1}{200 L} \right)$$

3. From inspection of the above diagram:

$$X + D = L/2 \text{ or } L = 2(X + D)$$

(Equation 26.4-21)

VERTICAL CURVE COMPUTATIONS

Figure 26.4H

By substituting $2(X+D)$ for L , and A for (G_2-G_1) into [Equation 26.4-8](#). Yields:

$$A X^2 + (-400 Z) X + (-400 DZ) = 0 \quad (\text{Equation 26.4-22})$$

4. Solve for "X" given the quadratic equation:

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{400Z \pm \sqrt{160\ 000} Z^2 + 1600 ADZ}{2A} \quad (\text{Equation 26.4-23})$$

Solving for X will result in two answers. If both answers are positive, there are two solutions. If one answer is negative, it can be eliminated and only one solution exists.

5. Substitute X and D into [Equation 26.4-21](#) and solve for L:

Note: Two positive X values, will result in two L solutions. Desirably, the longer vertical curve solution should be used provided it meets the stopping sight distance criteria (based on the selected design speed and algebraic difference in grades, see [Figures 26.4A](#) and [26.4C](#)).

VERTICAL CURVE COMPUTATIONS

(Continued)

Figure 26.4H

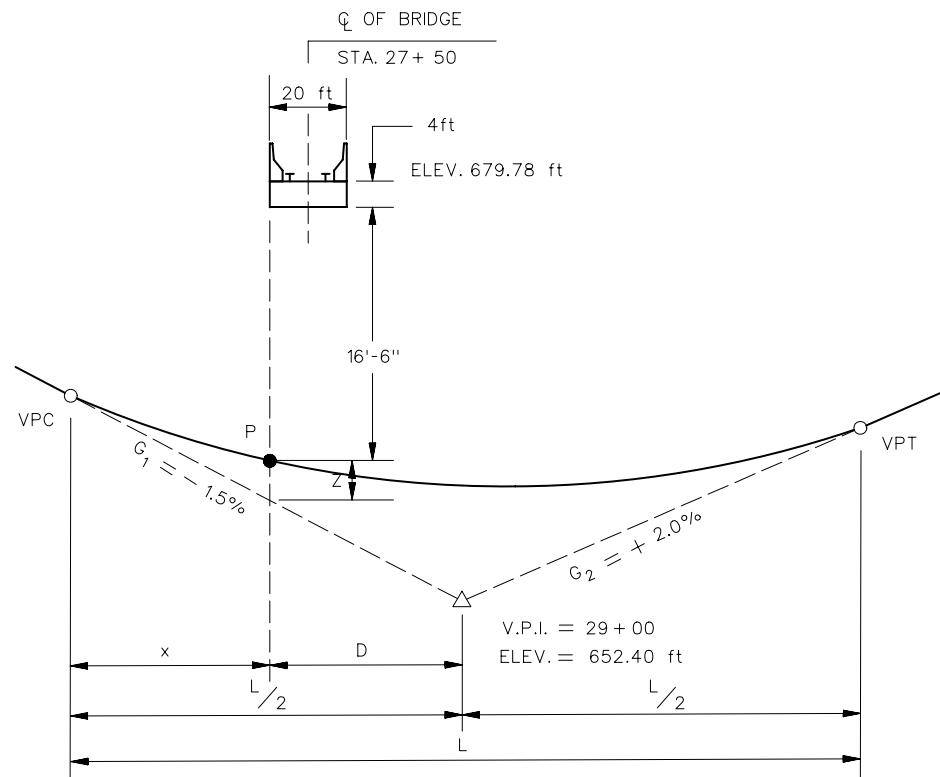
Example 26.4-2

Given: Design Speed = 55 mph
 $G_1 = -1.5\%$
 $G_2 = +2.0\%$
 $A = 3.5\%$
 VPI Station = 29 + 00.00
 VPI elevation = 652.40 ft

Problem: At Station 27 + 50, the new highway must pass under the center of an existing railroad which is at elevation 679.78 ft at the highway centerline. The railroad bridge that will be constructed over the highway will be 4 ft in depth, 20 ft in width and at right angles to the highway. What would be the length of the symmetrical vertical curve that would provide a 16 ft - 6 in clearance under the railroad bridge?

Solution:

1. Sketch the problem with known information labeled.



**VERTICAL CURVE COMPUTATIONS
(Example 26.4-2)**

Figure 26.4I

Example 26.4-2 (Continued)

2. Determine the station where the minimum 16 ft – 6 in vertical clearance will occur (Point P):

From inspection of the sketch, the critical location is on the left side of the railroad bridge. The critical station is:

$$\begin{aligned} \text{STA. P} &= \text{BRIDGE CENTERLINE STATION} - \frac{1}{2} (\text{BRIDGE WIDTH}) \\ \text{STA. P} &= \text{STA. } 27 + 50 - \frac{1}{2} (20 \text{ ft}) \\ \text{STA. P} &= \text{STA. } 27 + 40 \end{aligned}$$

3. Determine the elevation of Point P:

$$\begin{aligned} \text{ELEV. P} &= \text{ELEV. TOP RAILROAD BRIDGE} - \text{BRIDGE DEPTH} - \text{CLEARANCE} \\ \text{ELEV. P} &= 679.78 \text{ ft} - 4.0 \text{ ft} - 16.5 \text{ ft} \\ \text{ELEV. P} &= 659.28 \text{ ft} \end{aligned}$$

4. Determine distance, D, from Point P to VPI:

$$\begin{aligned} D &= \text{STA. VPI} - \text{STA. P} \\ &= (29 + 00) - (27 + 40) = 160 \text{ ft} \end{aligned}$$

5. Determine the tangent elevation at Point P:

$$\begin{aligned} \text{ELEV. TANGENT AT P} &= \text{ELEV. VPI} - G_1 \left(\frac{D}{100} \right) \\ &= 652.40 - (-1.5) \left(\frac{160}{100} \right) = 654.80 \text{ ft} \end{aligned}$$

6. Determine the vertical curve correction (Z) at Point P:

$$\begin{aligned} Z &= \text{ELEV. ON CURVE} - \text{ELEV. OF TANGENT} \\ &= 659.28 - 654.80 \\ &= 4.48 \text{ ft} \end{aligned}$$

7. Solve for X using [Equation 26.4-23](#):

$$X = \frac{400 Z \pm \sqrt{160\,000 Z^2 + 1600 ADZ}}{2A}$$

VERTICAL CURVE COMPUTATIONS
(Example 26.4-2)
(Continued)

Figure 26.4I

$$X = \frac{400(4.48) \pm \sqrt{(160000)(4.48^2) + 1600(3.5)(160)(4.48)}}{2(3.5)}$$

$$X = 640 \text{ ft} \quad \text{AND} \quad X = -128 \text{ ft} \quad (\text{Disregard})$$

8. Using [Equation 26.4-21](#), solve for L:

$$\begin{aligned} L &= 2(X + D) \\ L &= 2(640 + 160) \\ L &= 1600 \text{ ft} \end{aligned}$$

9. Determine if the solution meets the desirable stopping sight distance for the 55 mph design speed. From [Figure 26.4C](#), the design K-value:

$$K = 115$$

The algebraic difference in grades:

$$A = G_2 - G_1 = (+2.0) - (-1.5) = 3.5.$$

From Equation 26.4-5, the minimum length of vertical curve that meets the stopping sight distance:

$$\begin{aligned} L_{\text{MIN}} &= KA \\ L_{\text{MIN}} &= (115) 3.5 = 402.5 \text{ ft} \end{aligned}$$

L = 1600 ft will provide the design stopping sight distance.

VERTICAL CURVE COMPUTATIONS
(Example 26.4-2)
 (Continued)

Figure 26.4I